Fidelity susceptibility and general quench near an anisotropic quantum critical point

Victor Mukherjee[1,*] and Amit Dutta[1,†]

[1]Department of Physics, Indian Institute of Technology Kanpur, Kanpur 208 016, India

We study the scaling behavior of fidelity susceptibility density ($\chi_{\rm f}$) at or close to an anisotropic quantum critical point characterized by two different correlation length exponents $\nu_{||}$ and $\nu_{\perp}$ along parallel and perpendicular spatial directions, respectively. Our studies show that the response of the system due to a small change in the Hamiltonian near an anisotropic quantum critical point is different from that seen near an isotropic quantum critical point. In particular, for a finite system with linear dimension $L_{||}$ ($L_{\perp}$) in the parallel (perpendicular) directions, the maximum value of $\chi_{\rm f}$ is found to increases in a power-law fashion with $L_{||}$ for small $L_{||}$, with an exponent depending on both $\nu_{||}$ and $\nu_{\perp}$ and eventually crosses over to a scaling with $L_{\perp}$ for $L_{||}^{1/\nu_{||}} \gtrsim L_{\perp}^{1/\nu_{\perp}}$. We also propose scaling relations of heat density and defect density generated following a quench starting from an anisotropic quantum critical point and connect them to a generalized fidelity susceptibility. These predictions are verified exactly both analytically and numerically taking the example of a Hamiltonian showing a semi-Dirac band-crossing point.

PACS numbers: 64.70.qj,64.70.Tg,03.75.Lm,67.85.-d

I. INTRODUCTION

Recent studies on fidelity and fidelity susceptibility[1–4] (χ_F) near a quantum critical point have contributed to a deeper understanding of a quantum phase transition[5–7] from the viewpoint of quantum information theory. Fidelity is the measure of overlap of two neighbouring ground states of a quantum Hamiltonian in the parameter space. Fidelity susceptibility provides quantitatively the rate of change of the ground state under an infinitesimal variation of the parameters of the Hamiltonian. Since the ground state of a quantum many-body system exhibits different types of symmetries on either side of a quantum critical point[5] (QCP), a sharp drop of fidelity is observed right there. At the same time, the fidelity susceptibility usually diverges in a power law fashion with the system size where the exponent is given in terms of the quantum critical exponents[2–4,8–16]. In recent years a series of works have been directed to understanding the connection between fidelity susceptibility to quantum phase transition at critical[2–4,8–16] and multicritical points[17]. Studies of fidelity per site[4,18], reduced fidelity[4,19–21] and geometric phase, which is also closely related to fidelity susceptibility[2], near quantum critical[22,23] and multicritical[24] points have also been interesting areas of research.

In this paper, we extend the investigation on fidelity susceptibility to the case of an anisotropic quantum critical point (AQCP) and highlight the marked difference with the corresponding studies on an isotropic QCP. An interesting realization of an AQCP is seen in semi-Dirac band crossing points[25,26] where the energy gap scales linearly with momentum along one spatial direction but quadratically along others unlike Dirac points in graphene where a gap opens linearly along both the directions[27]. The possibility of such a semi-Dirac point has been reported recently[25,28] using a three unit cell slab of VO_2 confined within insulating TiO_2 and also in liquid He[3]. A series of works on low energy properties of a system with a semi-Dirac point has already been reported[26,29,30]. It is to be noted that the scaling of defect density following a slow quench across a QCP, namely the Kibble Zurek scaling[31–39], has also been generalized to an AQCP using a semi-Dirac Hamiltonian[40]. An AQCP can also be realized at the edge of the gapless region of a two dimensional Kitaev model in a honeycomb lattice[41,42] for which the Kibble Zurek scaling has also been proposed[43].

Let us consider a d-dimensional quantum mechanical Hamiltonian $H(\lambda)$ designated by a parameter λ. For two ground state wavefunctions $\psi_0(\lambda)$ and $\psi_0(\lambda+\delta\lambda)$ infinitesimally separated in the parameter space ($\delta\lambda \to 0$), we can define fidelity (F) as[1–4]

$$F = |\langle\psi_0(\lambda)|\psi_0(\lambda+\delta\lambda)\rangle| \approx 1 - \frac{\delta\lambda^2}{2}\chi_F(\lambda) + \cdots \qquad (1)$$

where the fidelity susceptibility $\chi_{\rm F}$ is the first non-vanishing term in the expansion of fidelity. The scaling behavior of $\chi_{\rm F}$ at a QCP is well established[2,4,13]. Let us choose the Hamiltonian to be of the form $H = H_0 + \lambda H_I$. Here H_0 is the Hamiltonian describing a QCP at $\lambda = 0$ while $H_I \equiv \partial_\lambda H\big|_{\lambda=0}$ is the perturbation not commuting with H_0. One can relate the fidelity susceptibility density $\left(\chi_{\rm f} = 1/L^d \chi_{\rm F}\right)$ to the connected imaginary time (τ) correlation function of the perturbation $H_I(\tau)$ using the relation[2]

$$\chi_{\rm f}(\lambda) = \frac{1}{L^d}\chi_{\rm F} = \frac{1}{L^d}\int_0^\infty \tau\langle H_I(\tau)H_I(0)\rangle_c d\tau. \qquad (2)$$

Using dimensional analysis in Eq. (2), we get that the scaling dimension of $\chi_{\rm f}$ is given by $\dim[\chi_{\rm f}] = 2\Delta_{H_I} - 2z + d$ where z is the dynamical exponent associated with the QCP and Δ_{H_I} is the scaling dimension of the operator H_I. Clearly a negative value of the scaling dimension leads to a fidelity susceptibility diverging with the system size L at the QCP as $\chi_{\rm f}(\lambda = 0) \sim L^{2z-d-2\Delta_{H_I}}$.

A positive value, on the other hand, implies a singular χ_f though the singular behavior appears as a subleading correction to a nonuniversal constant[11]. A marginal or relevant perturbation H_I (so that λH_I scales as the energy) allows us to make an additional simplification coming from $\Delta_{H_I} = z - 1/\nu$ so that at the critical point[8–11]

$$\chi_f \sim L^{2/\nu - d}. \tag{3}$$

Further we get a cross-over from system size dependence to λ dependence when the correlation length $\xi \sim \lambda^{-\nu}$ becomes of the order of system size:

$$\chi_f \sim |\lambda|^{\nu d - 2}. \tag{4}$$

These asymptotics are dominant for $d\nu < 2$ and subleading for $d\nu > 2$, while at $d\nu = 2$ there are additional logarithmic singularities[8,11].

In the following analysis, we show that the general scaling of fidelity susceptibility valid near an isotropic QCP gets modified due to the anisotropy in critical behavior. The changed scaling form naturally includes the correlation length exponents along the different spatial directions, namely $\nu_{||}$ and $\nu_\perp$. In addition, for a finite system with linear dimension $L_{||}$ ($L_\perp$) in the parallel (perpendicular) directions, the maximum value of χ_f increases with $L_{||}$ in the limit of small $L_{||}$ ($L_{||}^{1/\nu_{||}} \ll L_\perp^{1/\nu_\perp}$) only. In contrast, for higher values of $L_{||}$ ($L_{||}^{1/\nu_{||}} \gtrsim L_\perp^{1/\nu_\perp}$), we observe a crossover and χ_f scales with $L_\perp$. We also study the defect density, and heat density following a rapid quantum quench starting from an anisotropic quantum critical point and relate them through a generalized fidelity susceptibility[11]. We also highlight the connection to the Kibble-Zurek Scaling for the defect density following a slow quench through an AQCP and retrieve the scaling relations derived previously[40,43].

The paper is organized as follows: section II provides a general scaling relation of χ_f associated with an AQCP. We do also propose the same for the heat density and the defect density following a general quench starting from the AQCP, and relate them through a generalized fidelity susceptibility density. In section III we have taken a model Hamiltonian which shows an AQCP occurring in the physical systems described above and confirm our scaling predictions using exact analytical and numerical methods. Concluding remarks are presented in section IV.

II. GENERAL SCALING RELATIONS

A. Fidelity Susceptibility

Let us consider a d dimensional quantum Hamiltonian showing an AQCP at $\lambda = 0$. The correlations length exponent is $\nu = \nu_{||}$ along m spatial directions and $\nu = \nu_\perp$ along rest of the $(d - m)$ directions, called the parallel and perpendicular directions, respectively. The fidelity susceptibility as obtained from adiabatic perturbation theory[32,44] is of the form[4]

$$\chi_F = \sum_{n \neq 0} \frac{|\langle \psi_n | \frac{\partial H}{\partial \lambda} | \psi_0 \rangle|^2}{(E_n - E_0)^2}, \tag{5}$$

so that for a finite system with linear dimension $L_{||}$ ($L_\perp$) in the parallel (perpendicular) directions, the corresponding fidelity susceptibility density (χ_f) can be written as

$$\chi_f = \frac{1}{L_{||}^m L_\perp^{d-m}} \chi_F = \frac{1}{L_{||}^m L_\perp^{d-m}} \sum_{n \neq 0} \frac{|\langle \psi_n | \frac{\partial H}{\partial \lambda} | \psi_0 \rangle|^2}{(E_n - E_0)^2}. \tag{6}$$

Here, E_0 and E_n denote the energy of the ground state and nth energy level, respectively. We can contrast the above equation (6) with the specific heat density (the second derivative of the ground state energy density ($E_0/L_{||}^m L_\perp^{d-m}$)) given by[9]

$$\begin{aligned} \chi_E &= -\frac{1}{L_{||}^m L_\perp^{d-m}} \partial^2 E_0 / \partial \lambda^2 \\ &\sim \frac{1}{L_{||}^m L_\perp^{d-m}} \sum_{n \neq 0} \frac{|\langle \psi_n | \frac{\partial H}{\partial \lambda} | \psi_0 \rangle|^2}{E_n - E_0}. \end{aligned} \tag{7}$$

Comparison of Eqs. (6) and (7) indicates that near an AQCP, a much stronger divergence of χ_f as compared to χ_E is expected; this is due to the higher power of energy difference term in the denominator of χ_f.

We note that near an AQCP, the specific heat $\chi_E \sim |\lambda|^{-\alpha}$ where below the upper critical dimension the exponent α satisfies a modified hyperscaling relation[7,45,46] $2 - \alpha = \nu_{||} m + \nu_\perp (d - m) + \nu_{||} z_{||}$. In the limit of large $|\lambda|$ ($|\lambda| \gg L_{||}^{-1/\nu_{||}}, L_\perp^{-1/\nu_\perp}$), χ_E scales as

$$\chi_E \sim |\lambda|^{-\alpha} \sim |\lambda|^{\nu_{||} m + \nu_\perp (d-m) + \nu_{||} z_{||} - 2}. \tag{8}$$

Now, in the same limit the scaling of the fidelity susceptibility density is given by[9]

$$\begin{aligned} \chi_f &= \frac{1}{L_{||}^m L_\perp^{d-m}} \sum_{n \neq 0} \frac{|\langle \psi_n | \frac{\partial H}{\partial \lambda} | \psi_0 \rangle|^2}{(E_n - E_0)^2} \sim \frac{\chi_E}{|E_n - E_0|} \\ &\sim |\lambda|^{\nu_{||} m + \nu_\perp (d-m) - 2} \quad (|\lambda| \gg L_{||}^{-1/\nu_{||}}, L_\perp^{-1/\nu_\perp}). \end{aligned} \tag{9}$$

In deriving Eq. (9) we have used Eq. (8) and the fact that near the AQCP, $E_n - E_0 \sim |\lambda|^{\nu_{||} z_{||}} = |\lambda|^{\nu_\perp z_\perp}$. In the special case of $\nu_{||} = \nu_\perp = \nu$, we retrieve the expected scaling relation $\chi_f \sim \lambda^{\nu d - 2}$ valid near an isotropic quantum critical point[8,11] (see eq. (4)).

On the other hand, right at the AQCP ($\lambda = 0$), and in the limit $L_{||}^{1/\nu_{||}} \ll L_\perp^{1/\nu_\perp}$, $\chi_f(\lambda = 0)$ scales with the system size $L_{||}$ as

$$\chi_f(\lambda = 0) \sim L_{||}^{\frac{2}{\nu_{||}} - \frac{\nu_\perp}{\nu_{||}}(d-m) - m} \quad (L_{||}^{1/\nu_{||}} \ll L_\perp^{1/\nu_\perp}). \tag{10}$$

However, in the opposite limit $L_{||}^{1/\nu_{||}} \gg L_{\perp}^{1/\nu_{\perp}}$, $\chi_{\rm f}(\lambda=0)$ instead starts scaling with $L_{\perp}$, and Eq. (10) gets modified to

$$\chi_{\rm f}(\lambda=0) \sim L_{\perp}^{\frac{2}{\nu_{\perp}}-\frac{\nu_{||}}{\nu_{\perp}}m-(d-m)} \quad (L_{||}^{1/\nu_{||}} \gg L_{\perp}^{1/\nu_{\perp}}). \quad (11)$$

Clearly, the special condition $L_{||}^{1/\nu_{||}} \sim L_{\perp}^{1/\nu_{\perp}}$ yields

$$\chi_{\rm f}(\lambda=0) \sim L_{||}^{\frac{2}{\nu_{||}}-\frac{\nu_{\perp}}{\nu_{||}}(d-m)-m} \sim L_{\perp}^{\frac{2}{\nu_{\perp}}-\frac{\nu_{||}}{\nu_{\perp}}m-(d-m)}. \quad (12)$$

The above scalings in Eqs. (10 - 12) suggest $\chi_{\rm f}(\lambda=0)$ initially increases with $L_{||}$ until $L_{||}^{1/\nu_{||}} \sim L_{\perp}^{1/\nu_{\perp}}$. Beyond which $\chi_{\rm f}(\lambda=0)$ becomes independent of $L_{||}$ and saturates to a constant value. However, in this limit the fidelity susceptibility density scales with $L_{\perp}$, as shown in Eq. (11).

An alternative way of arriving at the above scalings is by the use of correlation functions[2–4,8–10,12]:

$$\chi_{\rm f} = \frac{1}{L_{||}^{m} L_{\perp}^{d-m}} \int_0^{\infty} \tau \langle H_I(\tau) H_I(0)\rangle_c d\tau, \quad (13)$$

where we define

$$H_I(\tau) = e^{H\tau} H_I e^{-H\tau}$$

and

$$\langle H_I(\tau)H_I(0)\rangle_c = \langle H_I(\tau)H_I(0)\rangle - \langle H_I(\tau)\rangle\langle H_I(0)\rangle,$$

with τ being the imaginary time. For a relevant perturbation λH_I should scale as the energy, so that $H_I \sim \lambda^{\nu_{||}z_{||}-1}$. Using the relation $\tau \sim L_{||}^{z_{||}}$ and $L_{||,\perp} \sim \lambda^{-\nu_{||,\perp}}$, we get the scaling of $\chi_{\rm f}$ from Eq. (13) given by

$$\chi_{\rm f} \sim |\lambda|^{\nu_{||}m+\nu_{\perp}(d-m)-2}, \quad (14)$$

which is identical to Eq. (9).

B. Heat and defect density following a sudden quench

In this section we study a sudden quench[47,48] of a quantum system of amplitude λ, starting from the AQCP. The quantities of interest are defect density[37–39] (n_{ex}) and heat density[11] (Q) generated in the process. Advantage of using heat density, or the excess energy above the new ground state, is that it can be defined even for non-integrable systems. On the other hand, for an integrable system with non-interacting quasi-particles, it is useful to define defect density, which is a measure of the density of excited quasi-particles generated in the system.

As λ is suddenly increased from $\lambda=0$ to its final value λ, all the momentum modes $k_{||} \lesssim \lambda^{\nu_{||}}$ and $k_{\perp} \lesssim \lambda^{\nu_{\perp}}$ get excited with excitation energy $\sim \lambda^{\nu_{||}z_{||}} = \lambda^{\nu_{\perp}z_{\perp}}$ for each mode. This gives an excitation energy density or heat density of the form

$$Q \sim \lambda^{\nu_{||}m+\nu_{\perp}(d-m)+\nu_{||}z_{||}}. \quad (15)$$

Defect density is related to the probability of excitation, which in turn can be expressed in terms of fidelity susceptibility[8,11,49]. Following the above argument one finds that

$$n_{ex} \sim \lambda^2 \chi_{\rm f} \sim \lambda^{\nu_{||}m+\nu_{\perp}(d-m)}. \quad (16)$$

Eq. (16) can also be derived by noticing that for a sudden quench of amplitude λ, all the momentum modes $k_1 \lesssim \lambda^{\nu_{||}}$ and $k_2 \lesssim \lambda^{\nu_{\perp}}$ get excited with unit probability, giving $n_{ex} \sim \lambda^{\nu_{||}m+\nu_{\perp}(d-m)}$.

C. Generalized fidelity susceptibility density

In this section we deal with a generic quench from an AQCP at time $t=0$ given by

$$\lambda(t) = \delta \frac{t^r}{r!}\Theta(t), \quad (17)$$

where δ is a small parameter, and Θ is the step function[11]. The case $r=0$ denotes a rapid quench of amplitude δ; the case $r=1$ implies a slow linear quench with a rate δ and so on. In all these cases the limit $\delta \to 0$ is considered to signify a slow adiabatic time evolution. If the system is initially in the ground state, the transition probability to the instantaneous excited state as obtained from the adiabatic perturbation theory is given by

$$\begin{aligned} P_{ex} &= \delta^2 \sum_{n\neq 0} \frac{|\langle\psi_n|\frac{\partial H}{\partial\lambda}|\psi_0\rangle|^2}{(E_n-E_0)^{2r+2}} \\ &= \delta^2 L_{||}^{m} L_{\perp}^{d-m} \chi_{2r+2}, \end{aligned} \quad (18)$$

which leads to a density of defect of the form

$$n_{ex} = \frac{1}{L_{||}^{m} L_{\perp}^{d-m}} P_{ex} = \delta^2 \chi_{2r+2}. \quad (19)$$

In the above, we have used the definition of a generalized fidelity susceptibility density χ_l given by[11]

$$\chi_l = \frac{1}{L_{||}^{m}} \frac{1}{L_{\perp}^{d-m}} \sum_{n\neq 0} \frac{|\langle\psi_n|\frac{\partial H}{\partial\lambda}|\psi_0\rangle|^2}{(E_n-E_0)^l}. \quad (20)$$

From Eq. (20), one finds that χ_1 stands for the specific density $\chi_{\rm E}$ while χ_2 is the fidelity susceptibility density $\chi_{\rm f}$; χ_4, on the other hand, yields the excitation probability following a slow linear quench starting from an AQCP.

In the same spirit as in Eq. (2), a general χ_l can also be expressed in terms of time dependent connected correlation functions given by

$$\chi_l = \frac{1}{L_{||}^{m} L_{\perp}^{d-m}(l-1)!} \int_0^{\infty} \tau^{l-1} \langle H_I(\tau)H_I(0)\rangle_c d\tau. \quad (21)$$

Now, using $\lambda \sim L_{||}^{-1/\nu_{||}} \sim L_{\perp}^{-1/\nu_{\perp}}$ and $t \sim L_{||}^{z_{||}} \sim L_{\perp}^{z_{\perp}}$ in Eq. (17) leads to the scaling relations $L_{||} \sim \delta^{-\frac{\nu_{||}}{1+\nu_{||}z_{||}r}}$,

$L_\perp \sim \delta^{-\frac{\nu_\perp}{1+\nu_\perp z_\perp r}} = \delta^{-\frac{\nu_\perp}{1+\nu_{||}z_{||}r}}$. These suggest that one can further conclude $H_I \sim \lambda^{\nu_{||}z_{||}-1} \sim \delta^{\frac{\nu_{||}z_{||}-1}{1+r\nu_{||}z_{||}}}$, and $\tau \sim L_{||}^{z_{||}} \sim \delta^{-\frac{\nu_{||}z_{||}}{1+\nu_{||}z_{||}r}}$. Substituting for $L_{||}$, $L_\perp$, H_I and τ in Eq. (21) with $l = 2r+2$ one gets in the limit $\delta \gg L_{||}^{-\frac{1}{\nu_{||}}-z_{||}r}, L_\perp^{-\frac{1}{\nu_\perp}-z_\perp r}$

$$\chi_{2r+2} \sim \delta^{\frac{\nu_{||}m+\nu_\perp(d-m)-2-2\nu_{||}z_{||}r}{1+\nu_{||}z_{||}r}}. \tag{22}$$

Therefore for a generic quench from an AQCP scaling of defect density gets modified to

$$n_{ex} \sim \delta^{\frac{\nu_{||}m+\nu_\perp(d-m)}{\nu_{||}z_{||}r+1}} \quad \left(\delta \gg L_{||}^{-\frac{1}{\nu_{||}}-z_{||}r}, L_\perp^{-\frac{1}{\nu_\perp}-z_\perp r}\right), \tag{23}$$

while the corresponding heat density scales as

$$Q \sim \lambda^{\nu_{||}z_{||}} n_{ex} \sim \delta^{\frac{\nu_{||}m+\nu_\perp(d-m)+\nu_{||}z_{||}}{\nu_{||}z_{||}r+1}}. \tag{24}$$

The expressions for n_{ex} and Q match exactly with the same for fast quench (Eqs. (15,16)) if we put $r = 0$ and $\delta = \lambda$, whereas, the case $r = 1$ correctly reproduces the values for a slow linear quench starting from the AQCP[40,43].

The scaling relations presented above are valid as long as the corresponding exponents do not exceed two. Otherwise contributions from short wavelength modes become dominant and hence the low energy singularities associated with the critical point become subleading[8,11].

III. MODEL AND HAMILTONIAN

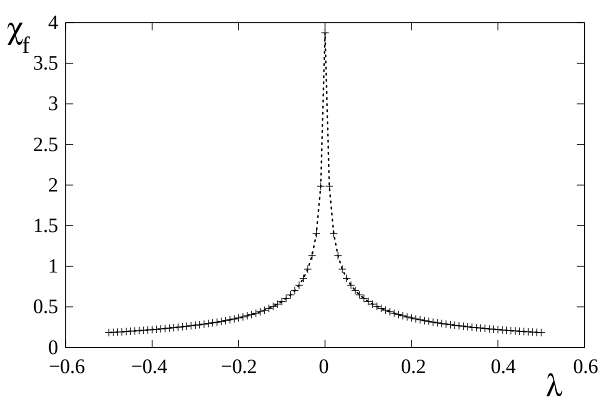

FIG. 1: Variation of χ_f with λ, as obtained numerically for $L_{||} = 10000$, $L_\perp = 1000$, $\nu_{||} = 1/2$, $\nu_\perp = 1$, $d = 2$ and $m = 1$. χ_f peaks at the AQCP, and falls as $|\lambda|^{-1/2}$, as predicted in Eq. (9).

We illustrate the above analytical predictions using the representative case of a semi-Dirac point in spatial dimension $d = 2$. In this case, $\nu_{||} = 1/2$, $\nu_\perp = 1$ and

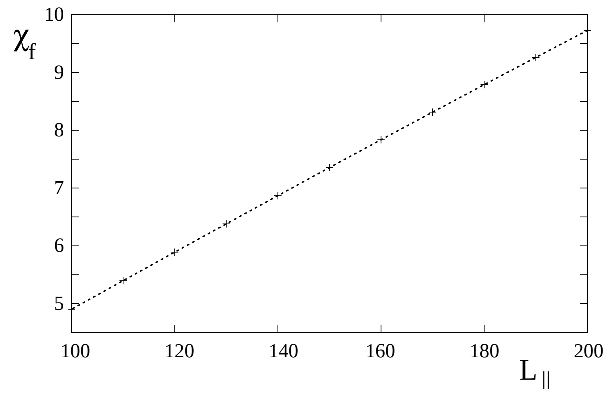

FIG. 2: Variation of $\chi_f(\lambda = 0)$ with $L_{||}$ as obtained numerically for $L_\perp = 100000$, $\nu_{||} = 1/2$, $\nu_\perp = 1$, $d = 2$ and $m = 1$. χ_f diverges as $\chi_f \sim L_{||}$, in agreement with the scaling given in Eq. (10).

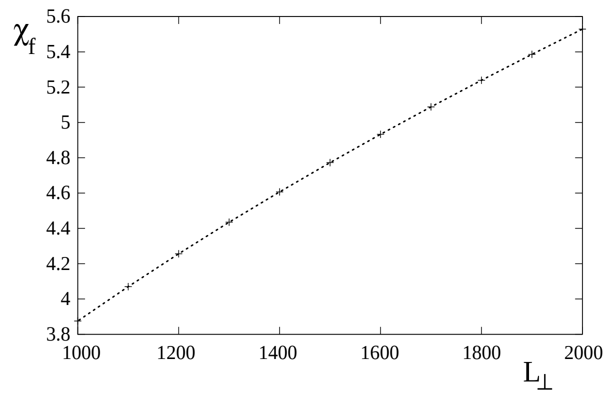

FIG. 3: Variation of $\chi_f(\lambda = 0)$ with $L_\perp$ as obtained numerically for $L_{||} = 10000$, $\nu_{||} = 1/2$, $\nu_\perp = 1$, $d = 2$ and $m = 1$. χ_f diverges as $\chi_f \sim L_\perp^{1/2}$ as expected from the scaling Eq. (11).

$d = 2, m = 1$. In the momentum k space the Hamiltonian near a semi-Dirac point can be written as the direct product of 2×2 Hamiltonians given by[26,40,43]

$$H_k = \begin{bmatrix} \lambda & k_{||}^2 + ik_\perp \\ k_{||}^2 - ik_\perp & -\lambda \end{bmatrix}. \tag{25}$$

The fidelity susceptibility density near the semi-Dirac point ($\lambda = 0$) can be written as as

$$\chi_f = \frac{1}{\pi^2}\int_{\pi/L_{||}}^{\pi}\int_{\pi/L_\perp}^{\pi} \frac{k_{||}^4 + k_\perp^2}{(\lambda^2 + k_{||}^4 + k_\perp^2)^2} dk_{||}dk_\perp. \tag{26}$$

Rescaling $k_{||}/\sqrt{\lambda} = x_1$, $k_\perp/\lambda = x_2$ and taking the limit $\lambda \gg L_{||}^{-2}, L_\perp^{-1}$ we get

$$\begin{aligned} \chi_f &= \frac{1}{|\lambda|^{1/2}\pi^2}\int_{\pi/\sqrt{\lambda}L_{||}}^{\pi/\sqrt{\lambda}}\int_{\pi/\lambda L_\perp}^{\pi/\lambda} \frac{x_1^4 + x_2^2}{\left(1 + x_1^4 + x_2^2\right)^2} dx_1 dx_2 \\ &\approx \frac{1}{|\lambda|^{1/2}\pi^2}\int_0^\infty\int_0^\infty \frac{x_1^4 + x_2^2}{\left(1 + x_1^4 + x_2^2\right)^2} dx_1 dx_2 \\ &\sim |\lambda|^{-1/2}, \end{aligned} \tag{27}$$

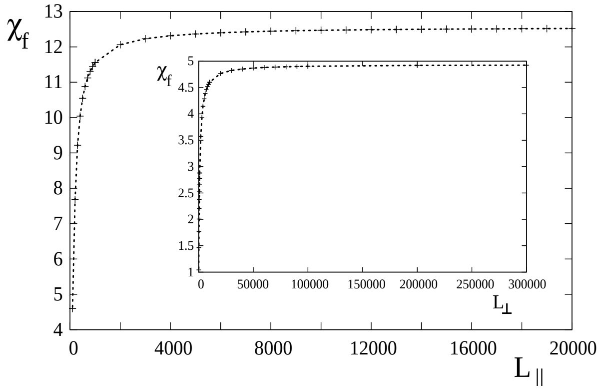

FIG. 4: Variation of $\chi_{\rm f}(\lambda = 0)$ with $L_{||}$ as obtained numerically for $\nu_{||} = 1/2$, $\nu_\perp = 1$, $d = 2$, $m = 1$ and $L_\perp = 10000$. $\chi_{\rm f}$ saturates at $L_{||}^2 \gtrsim L_\perp$, as expected from Eqs. (29, 30). Inset shows Variation of $\chi_{\rm f}$ with $L_\perp$ when $L_{||}$ kept fixed at $L_{||} = 100$. $\chi_{\rm f}$ saturates at $L_\perp \gtrsim L_{||}^2$.

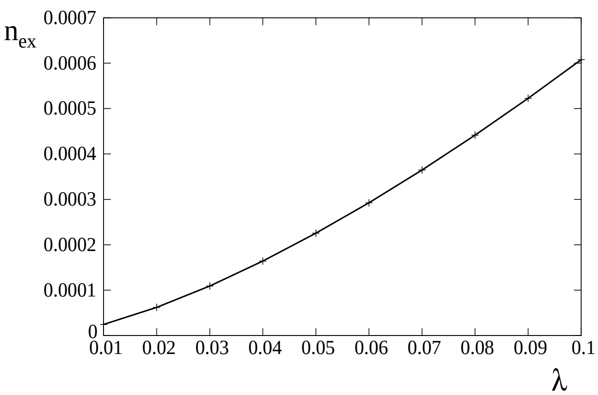

FIG. 5: Kink density n_{ex} as a function of λ as obtained numerically for $L_{||} = L_\perp = 1000$, $\nu_{||} = 1/2$, $\nu_\perp = 1$, $d = 2$, $m = 1$ and $\lambda \gg L_1^{-1/\nu_{||}}, L_\perp^{-1/\nu_\perp}$. n_{ex} varies as $n_{ex} \sim \lambda^{3/2}$, as predicted in Eq. (16).

which shows that divergence of $\chi_{\rm f}$ at $\lambda \to 0$ (see Fig. (1)) and exponent (1/2), and are in complete agreement with Eq. (9) for $d = 2$, $m = 1$, $\nu_{||} = 1/2$ and $\nu_\perp = 1$.

Right at the AQCP ($\lambda = 0$), we have.

$$\chi_{\rm f}(\lambda = 0) \;\approx\; \frac{1}{\pi^2} \int_{\pi/L_{||}}^{\infty} \int_{\pi/L_\perp}^{\infty} \frac{1}{k_{||}^4 + k_\perp^2} dk_{||} dk_\perp. \qquad (28)$$

The scalings $k_\perp = p k_{||}^2$ or $k_{||} = q\sqrt{k_\perp}$ simplify the above integral to

$$\begin{aligned} \chi_{\rm f} \;&=\; \frac{1}{\pi^2} \int_{\pi/L_{||}}^{\infty} \frac{dk_{||}}{k_{||}^2} \int_{\pi/L_\perp k_{||}^2}^{\infty} \frac{dp}{p^2+1} \\ &=\; \frac{1}{\pi^2} \int_{\pi/L_\perp}^{\infty} \frac{dk_\perp}{k_\perp^{3/2}} \int_{\pi/L_{||}\sqrt{k_\perp}}^{\infty} \frac{dq}{q^4+1}. \end{aligned} \qquad (29)$$

In the limit $L_{||}^2 \ll L_\perp$ (or $L_{||}^2 \gg L_\perp$) we can approximate $\pi/L_\perp k_{||}^2$ (or $\pi/L_{||}\sqrt{k_\perp}$) to zero, so that the scalings in Eq. (29) depends on one of the length scales, and we get

$$\begin{aligned} \chi_{\rm f}(\lambda = 0) \;&\sim\; L_{||} \qquad (\text{for } L_{||}^2 \ll L_\perp), \\ &\sim\; L_\perp^{1/2} \qquad (\text{for } L_{||}^2 \gg L_\perp). \end{aligned} \qquad (30)$$

Numerical verifications for the scalings of $\chi_{\rm f}$ with $L_{||}$ and $L_\perp$ discussed in Eq. (30) are provided in figures (2 - 4). Extending our analysis of $\chi_{\rm f}$ to find the defect density following a fast quench starting from the AQCP ($\lambda = 0$), we arrive at the scaling $n_{ex} \sim \lambda^2 \chi_{\rm f} \sim \lambda^{3/2}$. This relation is in perfect agreement with Eq. (16) and is verified numerically as shown in Fig. (5). However, scaling analysis of heat density Eq. (15) predicts $Q \sim |\lambda|^{2.5}$, which is subleading to the quadratic form $Q \sim \lambda^2$ arising from contributions of short wavelength modes. This leads to the scaling relation $Q \sim \lambda^2$. This quadratic scaling is also checked numerically in Fig. (6).

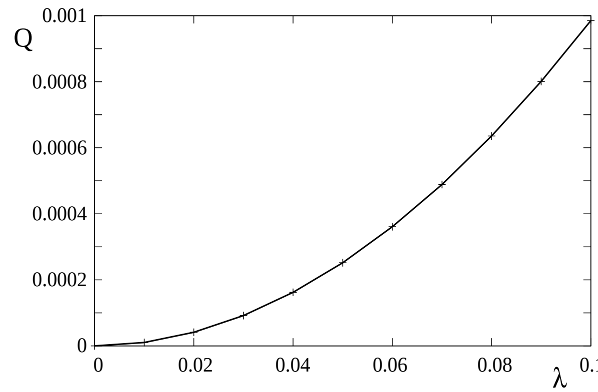

FIG. 6: Heat density Q as a function of λ as obtained numerically for $L_{||} = L_\perp = 1000$, $\nu_{||} = 1/2$, $\nu_\perp = 1$, $d = 2$, $m = 1$ and $\lambda \gg L_1^{-1/\nu_{||}}, L_\perp^{-1/\nu_\perp}$. Q follows the perturbative scaling law $Q \sim \lambda^2$, as discussed in the text.

IV. CONCLUSIONS

We have studied the scaling behavior of fidelity susceptibility near an anisotropic quantum critical point. Anisotropic critical behaviour modifies the general scaling form of $\chi_{\rm f}$. In particular, both $\nu_{||}$ and $\nu_\perp$ appear in the scaling. In addition, even though the maximum value of $\chi_{\rm f}$ scales with $L_{||}$ in the limit of small $L_{||}$, at higher values of the same a cross-over is observed and $\chi_{\rm f}$ starts scaling with $L_\perp$ instead. We also propose the scaling relations for the defect density and heat density following a generic quantum quench starting from an AQCP and relate them through a generalized fidelity susceptibility. We have verified our general scaling predictions both numerically and analytically using the illustrative example of a Hamiltonian showing a semi-Dirac point. Interestingly, we show that the heat density following a rapid quench starting from a two-dimensional semi-Dirac point varies quadratically with the amplitude and the scaling

arising due to low-energy critical modes appear only as a sub-leading correction.

Acknowledgments

The authors acknowledge U. Divakaran, A. Polkovnikov and R. R. P. Singh for collaboration in related works. A.D. acknowledges CSIR, New Delhi, for partial financial support.

* Electronic address: victor@iitk.ac.in
† Electronic address: dutta@iitk.ac.in

1. P. Zanardi, N. Paunkovic, Phys. Rev. E, **74**, 031123 (2006).
2. L. C. Venuti and P. Zanardi, Phys. Rev. Lett. **99**, 095701 (2007).
3. P. Zanardi, P. Giorda and M. Cozzini, Phys. Rev. Lett. **99**, 100603 (2007).
4. S.-J. Gu, Int. J. Mod. Phys. B **24**, 4371 (2010).
5. S. Sachdev, *Quantum Phase Transitions* (Cambridge University Press, Cambridge, 1999).
6. B. K. Chakrabarti, A. Dutta and P. Sen, *Quantum Ising Phases and transitions in transverse Ising Models*, m41 (Springer, Heidelberg,1996).
7. M. A. Continentino, *Quantum Scaling in Many-Body Systems* (World Scientific, 2001).
8. V. Gritsev and A. Polkovnikov arXiv:0910.3692, published in "Understanding Quantum Phase Transitions", edited by Lincoln D. Carr (Taylor and Francis, Boca Raton, 2010).
9. A. F. Albuquerque, F. Alet, C. Sire and S. Capponi, Phys. Rev. B **81**, 064418 (2010).
10. D. Schwandt, F. Alet and S. Capponi, Phys. Rev. lett., **103**, 170501 (2009).
11. C. De Grandi, V. Gritsev and A. Polkovnikov, Phys. Rev. B **81**, 012303 (2010).
12. S.-J. Gu and H.-Q. Lin, Euro. Phys. Lett. **87** (2009) 10003;
13. W.-L. You, Y.-W. Li, and S.-J. Gu, Phys. Rev. E **76**, 022101 (2007); S. Chen, L. Wang, S.-J. Gu, and Y. Wang, Phys. Rev. E **76**, 061108 (2007); S.-J. Gu, H.-M. Kwok, W.-Q. Ning, and H.-Q. Lin, Phys. Rev. B **77**, 245109 (2008); S. Chen, L. Wang, S.-J. Gu, and Y. Wang, Phys. Rev. A **77**, 032111 (2008); S. Yang, S.-J. Gu, C.-P. Sun, and H.-Q. Lin, Phys. Rev. A **78**, 012304 (2008); W.-C. Yu, H.-M. Kwok, J. Cao1, and S.-J. Gu, Phys. Rev. E **80**, 021108 (2009); Z. Wang, T. Ma, S.-J. Gu, and H.-Q. Lin, Phys. Rev. A **81**, 062350 (2010); .
14. N. T. Jacobson, S. Garnerone, S. Haas, and P. Zanardi, Phys. Rev. B **79**, 184427 (2009); A. T. Rezakhani, D. F. Abasto, D. A. Lidar, P. Zanardi,Phys. Rev. A **82**, 012321 (2010); S. Garnerone, D. Abasto, S. Haas, and P. Zanardi, Phys. Rev. A **79**, 032302 (2009); S. Garnerone, D. Abasto, S. Haas, and P. Zanardi, Phys. Rev. Lett. **102**, 057205 (2009); X.-M. Lu, Z. Sun, X. Wang, and P. Zanardi, Phys. Rev. A **78**, 032309 (2008); L. Venuti, H. Saleur, and P. Zanardi, Phys. Rev. B **79**, 092405 (2009); D. Abasto, A. Hamma, and P. Zanardi, Phys. Rev. A **78**, 010301(R) (2008); M. Cozzini, R. Ionicioiu, and P. Zanardi, Phys. Rev. B **76**, 104420 (2007); M. Cozzini, P. Giorda, and P. Zanardi, Phys. Rev. B **75**, 014439 (2007); L. Campos Venuti, M. Cozzini, P. Buonsante, F. Massel, N. Bray-Ali, and P. Zanardi, Phys. Rev. B **78**, 115410 (2008).
15. M. M. Rams and B. Damski, Phys. Rev. Lett. **106**, 055701 (2011).
16. H.-Q. Zhou, and J. P. Barjaktarevic, J. Phys. A: Math. Theor. **41**, 412001 (2008); H.-Q. Zhou, R. Ors, and G. Vidal, Phys. Rev. Lett. **100**, 080601 (2008); B. Li, S.-H. Li, and H.-Q. Zhou, Phys. Rev. E **79**, 060101(R) (2009); J.-H. Zhao and H.-Q. Zhou, Phys. Rev. B **80**, 014403 (2009); H.-Q. Zhou, J.-H. Zhao, B. Li, J. Phys. A: Math. Theor. **41**, 492002 (2008).
17. V. Mukherjee, A. Polkovnikov, and A. Dutta, Phys. Rev. B **83**, 075118 (2011).
18. H. -Q. Zhou, J. H. Zhao, and B. Li, J. Phys. A: Math. Theor. **41**, 492002(2008); J.-H. Zhao and H.-Q. Zhou, Phys. Rev. B **80** 014403 (2009).
19. M. Žnidarič and T. Prosen, J. Phys. A: Math. Gen. **36**, 2463(2003).
20. J. Ma, L. Xu, H.-N. Xiong, and X. Wang, Phys. Rev. E **78** 051126 (2008).
21. E. Eriksson and H. Johannesson, Phys. Rev. A **79** 060301(R) (2009).
22. A. C. M. Carollo and J. K. Pachos, Phys. Rev. Lett. **95**, 157203 (2005).
23. S.-L. Zhu, Phys. Rev. Lett. **96** 077206 (2006).
24. A. Patra, V. Mukherjee and A. Dutta, J. Stat. Mech. (2011) P03026.
25. V. Pardo and W. E. Pickett, Phys. Rev. Lett. **102**, 166803 (2009).
26. S. Banerjee, R. R. P. Singh, V. Pardo, and W. E. Pickett, Phys. Rev. Lett. **103**, 016402 (2009).
27. A. H. Castro Neto et. al, Rev. Mod. Phys. **81**, 109 (2009).
28. G. I. Volovik, JETP Lett. **73** (2001) 162.
29. V. Pardo, and W. E. Pickett, Phys. Rev. B **81**, 035111 (2010).
30. P. Delplace and G. Montambaux, Phys. Rev. B **82**, 035438 (2010).
31. W. H. Zurek, U. Dorner and P. Zoller, Phys. Rev. Lett. **95**, 105701 (2005)
32. A. Polkovnikov, Phys. Rev. B **72**, 161201(R) (2005).
33. J. Dziarmaga, Phys. Rev. Lett. **95**, 245701 (2005).
34. R. W. Cherng and L. S. Levitov, Phys. Rev. A **73**, 043614 (2006).
35. V. Mukherjee, U. Divakaran, A. Dutta and D. Sen, Phys. Rev. B **76**, 174303 (2007).
36. D. Sen, K. Sengupta, and S. Mondal, Phys. Rev. Lett. **101**, 016806 (2008).
37. J. Dziarmaga, Advances in Physics **59**, 1063 (2010).
38. A. Dutta, U. Divakaran, D. Sen, B. K. Chakrabarti, T. F. Rosenbaum and G. Aeppli, arXiv:1012.0653 (2010).
39. A. Polkovnikov, K. Sengupta, A. Silva, M. Vengalattore, arXiv:1007.5331 (2010).
40. A. Dutta, R.R.P. Singh, and U. Divakaran, EPL, **89** (2010) 67001.

[41] A. Kitaev, Ann. Phys. (N.Y.) **303**, 2 (2003).
[42] K. Sengupta, D. Sen, and S. Mondal, Phys. Rev. Lett. **100**, 077204 (2008).
[43] T. Hikichi, S. Suzuki, and K. Sengupta, Phys. Rev. B **82**, 174305 (2010).
[44] G. Rigolin, G. Ortiz, and V. H. Ponce, Phys. Rev. A 78, 052508 (2008).
[45] K. Binder and J.-S. Wang, J. Stat. Phys **55**, 87 (1989).
[46] A. Dutta, B. K. Chakrabarti, and J. K. Bhattacharjee, Phys. Rev. B **55**, 5619 (1997)
[47] D. Rossini, A. Silva, G. Mussardo and G. E. Santoro, Phys. Rev. Lett. **102**, 127204 (2009).
[48] E. Canovi, D. Rossini, R. Fazio, G. E. Santoro and A. Silva, Phys. Rev. B **83**, 094431 (2011).
[49] S.-J. Gu, Phys. Rev. E **79**, 061125 (2009).